

## **UNIVERSITY ENTRANCE EXAMINATION MATHEMATICS SYLLABUS**

There are two sections in the Mathematics syllabus. Students offering Mathematics at a standard equivalent to 'AO' level in the GCE Advanced Level Examination must study Section A.

Students offering Mathematics at a standard equivalent to 'A' level Mathematics in the GCE Advanced Level Examination must study both Section A and B.

# MATHEMATICS

## Syllabus for Section A

### SYLLABUS

1. Elementary two-dimensional cartesian coordinate geometry: distance between two points, the gradient and the equation of a straight line. Condition of two lines to be parallel or perpendicular.

2. Function. Inverse of a one-one function. Domain and range of a function. Composite functions. Graphical illustration of relationship between a function and its inverse.

Determination of unknown constants in a relationship by plotting an appropriate straight line graph.

3. Representation of a curve by means of a pair of parametric equations; equation of tangent and normal.

4. Elementary properties of quadratic equations. Range of values of the function  $ax^2 + bx + c$  by graphical and other methods.

Solution of quadratic inequalities.

5. Logarithms and surds, rational indices.

The remainder and factor theorems.

Factors of polynomials

Algebraic operations on polynomials and rational functions.

Partial fractions.

### NOTES

The notations  $y = f(x) = \sin x$  and  $f: x \rightarrow \sin x$  will be used.

Single parameter only. Conversion from parametric to cartesian coordinates and vice versa.

Including polynomials up to degree 3.

The include denominators such as  $(ax + b)(cx + d)(ex + f)$ ,  $(ax + b)(cx + d)^2$ , and  $(ax + b)(x^2 + c^2)$ .

6. Simultaneous equations, at least one linear, in two unknowns.

7. Elementary sequences Arithmetic and geometric progressions and their sum to  $n$  terms.

8. Binomial expansion of  $(a + b)^n$  for positive integral  $n$  and its use for simple approximations

The use of the expansion of  $(1 + x)^n$  when  $n$  is rational and  $|x| < 1$ .

9. Circular measure: arc length, area of a sector of a circle.

10. Trigonometric ratios of angles of any magnitude and their relationships. Graphs of sine, cosine and tangent.

The general solution of simple trigonometric equations involving any of the six simple trigonometric functions, including graphical interpretation.

The solution of triangles and determination of area.

Simple identities.

Addition formulae,

$\sin(A \pm B)$ ,  $\cos(A \pm B)$ ,  $\tan(A \pm B)$ ,

and application to multiple angles.

Expression of  $a \cos \theta + b \sin \theta$  as  $R \cos(\theta \pm \alpha)$  or  $R \sin(\theta \pm \alpha)$  and

solution of  $a \cos \theta + b \sin \theta = c$ .

11. Vectors in two dimensions: Magnitude of a vector, addition and subtraction of vectors, Multiplication by scalars. Displacement and position vectors, Unit vectors. Scalar product and its use to determine the angle between two lines. The ratio theorem  $\lambda OP + \mu OQ = (\lambda + \mu) OR$ .

Including the sum to infinity of geometric series.

Include the  $\Sigma$  notation.

Questions on the greatest term and on properties of the coefficients will not be asked.

To include the notation  $n!$  with  $0! = 1$ , and

$$\binom{n}{r}$$

Only the sine and cosine formulae and  $\frac{1}{2}bc \sin A$  will be needed, proofs not required.

Questions may be set using any vector notation including the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ .

<p>12. Simple properties and graphs of the logarithmic and exponential functions. Laws of logarithms. Solution of <math>a^x = b</math>.</p>	<p>Including <math>\ln x</math> and <math>e^x</math>. Their series expansions are not required. The definition <math>a^x = e^{x \ln a}</math></p>
<p>13. Differentiation.</p> <p>Derivative of <math>x^n</math> for <math>n</math> a rational number.</p> <p>Differentiation of standard functions, including products, quotients, composite functions and of simple functions defined implicitly or parametrically.</p> <p>Applications to gradients, tangents and normals, stationary points, velocity and acceleration, rates of change, small increments and approximations; practical problems involving maxima and minima.</p>	<p>Both <math>f'(x)</math> and <math>dy/dx</math> will be used.</p> <p>Formal proofs will not be required.</p> <p>Simple algebraic and trigonometric functions, <math>\ln x</math>, <math>e^x</math>, <math>a^x</math>, <math>\log_a x</math> but excluding inverse trigonometric functions.</p> <p>Any method of discrimination between stationary points will be acceptable.</p>
<p>14. Integration as the inverse of differentiation. Definite integrals. Integration of simple functions. Applications to plane areas, volumes, and problems in kinematics.</p>	<p>The integrals of <math>(ax + b)^n</math>, for <math>n</math> any rational number, <math>e^{ax+b}</math>, <math>\sin(ax + b)</math>, <math>\cos(ax + b)</math>. Excluding integration by parts or by substitution.</p>
<p>15. Elementary permutations and combinations.</p>	
<p>16. Introduction to probability. The sum and product laws of probability. Expectation.</p>	<p>Including the treatment of mutually exclusive and independent events.</p>
<p>17. Kinematics of a particle moving in a straight line with uniform acceleration.</p>	<p>To include use of <math>x - t</math> and <math>v - t</math> graphs.</p>

Syllabus for Section B

**Option (a): Pure Mathematics**

**SYLLABUS**

1. More difficult problems on topics defined in paragraphs 1 to 14 in Syllabus for Section A.
2. The approximations  $\sin x \cong x$ ,  $\tan x \cong x$ ,  $\cos x \cong 1 - \frac{1}{2} x^2$ .
3. The manipulation of simple algebraic inequalities. The modulus function.
4. The method of induction.
5. Curve sketching and equations in cartesian form including the following:  $y = f(x)$  and  $y^2 = f(x)$ .
6. Polar co-ordinates; sketching simple curves (for  $0 \leq \theta < 2\pi$  or  $-\pi < \theta \leq \pi$  or a subset of either of these) and  $r \geq 0$ . Area of a sector.
7. Complex numbers: algebraic and trigonometric forms; modulus and argument; complex conjugate; sum, product and quotient of two complex numbers.

**NOTES**

To include solutions reducible to the form  $f(x) > 0$ , where  $f(x)$  can be expressed in factors, and sketching the graph of  $y = f(x)$  in these cases.

Problems set may involve the summation of finite series.

Curves such as  $y = kx^n$  for simple rational  $n$ ,  $ax + by = c$ ,  $(x^2/a^2) + (y^2/b^2) = 1$ .

Knowledge of the effect of simple transformations, on the graph of  $y = f(x)$  as represented by  $y = af(x)$ ,  $y = f(x) + a$ ,  $y = f(x - a)$ ,  $y = f(ax)$ .

The relation of the equation of a graph to its symmetries.

Determination of asymptotes parallel to the axes is required.

Conversion from Cartesian to polar form (or vice versa) in simple cases is included.

The terms 'real part' and 'imaginary part' should be known.

The relation  $z\bar{z} = |z|^2$  should be known.

Representation of complex numbers in an Argand diagram, simple loci. De Moivre's theorem for an integral exponent (without proof); use of the relation  $e^{i\theta} = \cos \theta + i \sin \theta$ ; simple applications.

8. Expression of the coordinates (for position vector) of a point on a curve in terms of a parameter.

Vectors in three dimensions: unit vectors, the expression  $a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$  for a vector  $\mathbf{a}$  in terms of cartesian components; use of the scalar product  $\mathbf{a}\cdot\mathbf{b}$  in both the forms  $|\mathbf{a}||\mathbf{b}|\cos\theta$  and  $a_1b_1 + a_2b_2 + a_3b_3$ . Geometrical applications of scalar products; the equation of a plane in the form  $\mathbf{r}\cdot\mathbf{n} = p$ ; vector equation of a line in the form  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ .

The cartesian equations of lines and planes are also required. The angle between a line and a plane, between two planes, between two vectors or skew lines. Length of a projection. Distance from a point to a line or from a point to a plane.

9. Basic properties of a linear (vector) space; subspaces spanned by a set of vectors.

Linear dependence and independence.

Row space and column space of a matrix; the basis of a subspace.

Matrices as linear transformations  $\mathbb{R}^n \rightarrow \mathbb{R}^m$ .

Applications involving the expression of  $\sin(n\theta)$  or  $\cos(n\theta)$  in terms of sines and cosines are for  $n = 1, 2, 3$  only.

Unit vectors  $\mathbf{i}, \mathbf{j}, \mathbf{k}$ .

The properties:

(i)  $\mathbf{a}\cdot(\mathbf{l}\mathbf{b}) = \mathbf{l}(\mathbf{a}\cdot\mathbf{b})$

(ii)  $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$

(iii)  $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c}) = \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\cdot\mathbf{c}$

may be assumed.

Spaces are limited to those over the field  $\mathbb{R}$  only.

The ability to evaluate a  $3 \times 3$  determinant and to obtain the inverse of a  $3 \times 3$  matrix is expected.

Knowledge of rank and dimension is expected.

An elementary treatment only is expected but knowledge of range space and null space is required and this includes the nature of the solution of a set of linear equations.

Eigenvalues and eigenvectors of $2 \times 2$ and $3 \times 3$ matrices.	Their use in expressing matrices in the form $QDQ^{-1}$ where D is a diagonal matrix of eigenvalues. Applications of this process.
<p>10. The idea of a limit and the derivative defined as a limit. The gradient of a tangent as the limit of the gradient of a chord.</p> <p>Differentiation of inverse functions.</p> <p>Integration by parts and by substitution.</p> <p>The idea of area under a curve as the limit of a sum of areas of rectangles.</p> <p>Application to volumes of revolution.</p>	
11. Numerical integration, the trapezium rule.	Simple graphical consideration of the sign of the error of the trapezium rule is required.
12. Solution of first order differential equations by separating the variables; the reduction of a given differential equation to one of the type by means of a given simple substitution.	No particular knowledge of scientific, economics or other laws will be assumed, but mathematical formulation of given information may be required. Sketching of members of a family of solution curves. Knowledge of the terms 'complementary function' and 'particular integral'.

## Option (b): Particle Mechanics

### SYLLABUS

1. More kinematics: Newton's laws of motion, dynamics of particle moving in a straight line under forces expressed as functions of time or velocity of distance.
2. Forces treated as vectors, composition and resolution of forces on a particle. Triangle and polygon of forces. Equilibrium of particles under coplanar forces. Friction.
3. Simple cases of motion of two connected particles e.g. involving a single fixed smooth light pulley.
4. Elastic strings and springs, Hooke's law.
5. Kinetic and potential energy, work, power. Conservation of energy.  
  
Momentum and impulse; simple examples of conservation of momentum for particles moving in a straight line.
6. Simple cases of motion of a projectile.
7. Uniform circular motion.

### NOTES

Acceleration in the forms  $d^2x/dt^2$ ,  $dv/dt$ ,  $v dv/dx$ . Problems set will require only the solution of first order differential equations with separable variables.

Proofs of fundamental theorems are not required. Clearly drawn force diagrams and use of equations obtained by resolving the forces are required. Graphical solutions may be used in appropriate cases.

Gravitational and elastic potential energy.

Excluding coefficient of restitution.

Including cartesian parametric equations (parameter  $t$ ) or the  $(x, y)$  equation of the trajectory. Excluding properties of a parabola, range on an inclined plane.

Proof of formula for acceleration towards the centre is not required.



## Option (c): Probability and Statistics

### SYLLABUS

1. Graphical representation of numerical data.  
Various types of diagrams for continuous or discrete variables, either grouped or ungrouped.
2. Measures of average: mean, median, mode, modal class.  
Measures of dispersion: range, interquartile range, standard deviation, and variance.
3. Cumulative frequency tables and cumulative frequency curves (ogives).
4. Theoretical and empirical interpretations of probability; the basic probability laws including  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ; mutually exclusive events; conditional probability; independent events:  
$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B).$$
5. Probability distributions for discrete random variables, expectation, variance. Uniform, binomial and Poisson distributions; their means and variances (without proofs). The Poisson distribution as an approximation to the binomial distribution (without proof).

### NOTES

Including pie chart, bar chart, frequency polygon and histogram.

Effect on mean and S.D. of adding a constant to each observation and multiplying each observation by a constant.

Including finding of median, quartiles and percentiles from a cumulative frequency curve and by linear interpolation from a cumulative frequency table.

Problems will involve simple applications of the basic laws, but not the general form of Bayes' theorem. Tree diagrams. Venn diagrams and/or Karnaugh maps may be used but none of these methods will be specifically required in the problems set.

Simple examples may be set in which the distribution has to be found. knowledge and use of

$$E(aX + b) = aE(X) + b,$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2,$$

$$\text{Var}(aX + b) = a^2\text{Var}(X) \text{ is required.}$$

6. Probability distribution for continuous random variables; probability density function  $f$ ; expectation, variance; cumulative distribution function  $F$ : uniform (rectangular) distribution; the normal distribution excluding detailed analytical treatment; use of tables; the normal distribution as an approximation to the binomial distribution (without proof).

Problems may be set in which  $f(x)$  or  $F(x)$  are given or have to be found. Candidates are expected to be able to interpret the graphs of  $f(x)$  and  $F(x)$ , to know the relationship

$f(x) = F'(x)$ , and to calculate expectations, medians, modes, and expectations of simple functions of  $X$ . No detailed knowledge of bivariate distributions is required but the following results (without proof) should be known:

$$E(aX + bY) = aE(X) + bE(Y);$$

$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$  for independent variables  $X, Y$ ; if  $X, Y$  are independent normal variables, then  $aX + bY$  is normal variable.

7. General ideas of sampling methods and distributions; approximate normality of the distribution of sample mean (the central limit theorem, without proof). Estimation of population parameters from a sample; unbiasedness; confidence limits for population proportion and for the mean from large samples.

Sampling of attributes is included. Detailed knowledge of particular sampling methods is not expected, but candidates may be asked to comment on or answer simple probability questions on a given method of sampling.

Use of  $E(X) = \mu$ ,  $\text{Var}(X)$  or  $\sigma^2/n$ . Candidates are expected to know that an unbiased estimate of the population variance from a sample of size  $n$  is

$$(n-1)^{-1} \sum (x - \bar{x})^2.$$

## **RECOMMENDED TEXTS AND REFERENCES:**

### For Section A

- (1) Cheah Tat Huat et al: Additional Mathematics: Volume I, Oxford University Press (Singapore), 1986.
- (2) Teh Keng Seng: Pure Mathematics & Statistics (O-Level), Shing Lee Publishers, 1989.

### For Section A and Section B

- (1) L. Bostock & S. Chandler: Mathematics – The Core Course for A-Level, Stanley Thornes, (ELBS Edition) 1985.  
  
L. Bostock & S. Chandler: Mathematics – Mechanics & Probability, Stanley Thornes (ELBS Edition) 1985.
- (2) J. Crawshaw and J. Chambers: A Concise Course in A-Level Statistics with Worked Examples, Stanley Thornes, 1984.
- (3) Ho Soo Thong et al: College Mathematics: Volume I, Pan Pacific Publication, 1989.  
  
Y.M. Chow et al: College Mathematics: Volume II, Pan Pacific Publication, 1988.
- (4) C. Plumpton: New Tertiary Mathematics, Volume 1, Part 1 – Basic Pure Mathematics, Pergamon, 1980.  
  
C. Plumpton: New Tertiary Mathematics, Volume 1, Part 2 – Basic Applied Mathematics, Pergamon, 1980.