UNIVERSITY ENTRANCE EXAMINATION 2018

MATHEMATICS ('AO' LEVEL EQUIVALENT)

Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper has TWO (2) sections – A and B, and comprises **TEN (10)** printed pages.
2. Attempt all sections.
3. Answer all questions in section A. Indicate your answers on the answer paper provided. Each question carries 2 marks. Marks will not be deducted for wrong answers.
4. Answer not more than FIVE (5) questions from Section B. Write your answers on the answer paper provided. Begin each question on a fresh sheet of paper. Write the question number clearly. Each question carries 12 marks.
5. A non-programmable scientific calculator may be used. However, candidates should lay out systematically the various steps in the calculation.
6. At the end of the examination, attach the cover paper on top of your answer script. Complete the information required on the cover page and tie the papers together with the string provided. The colour of the cover paper for this examination is **GREEN**.
7. Do not take any paper, including the question paper and unused answer paper, out of the examination hall.

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SECTION A (40 Marks)

Answer all questions in this section. Each question carries 2 marks.

1. The graph of the function $f(x) = x^4 + \cos x$ is symmetric about

   (A) the x-axis

   (B) the origin

   (C) the y-axis

   (D) the line $y = x$

   (E) none of the above

2. Water is poured into a cylindrical container of radius 4 cm at the rate of $4\pi\,\text{cm}^3\text{s}^{-1}$. The rate at which the water level is rising is

   (A) $\frac{1}{4}\,\text{cm}\,\text{s}^{-1}$

   (B) $\frac{1}{2}\,\text{cm}\,\text{s}^{-1}$

   (C) $2\,\text{cm}\,\text{s}^{-1}$

   (D) $4\,\text{cm}\,\text{s}^{-1}$

   (E) none of the above

3. If
$$\tan x + \sec x = \frac{2}{7},$$
   then the value of $\tan x - \sec x$ is

   (A) $\frac{5}{7}$

   (B) $-\frac{5}{7}$

   (C) $\frac{7}{2}$

   (D) $-\frac{7}{2}$

   (E) none of the above

4. ABC is a triangle in which $AB = 9\,\text{cm}$, $BC = 40\,\text{cm}$ and $AC = 41\,\text{cm}$. The area of triangle ABC is

   (A) $180\,\text{cm}^2$

   (B) $184.5\,\text{cm}^2$

   (C) $360\,\text{cm}^2$

   (D) $820\,\text{cm}^2$

   (E) none of the above

5. The derivative of

$$\sqrt{\ln \sqrt{x}}$$

   with respect to x is

   (A) $\dfrac{1}{2\sqrt{x}\sqrt{\ln x}}$

   (B) $\dfrac{1}{2\sqrt{2}\sqrt{x}\sqrt{\ln x}}$

   (C) $\dfrac{1}{2x\sqrt{\ln x}}$

   (D) $\dfrac{1}{2x\sqrt{2}\,\sqrt{\ln x}}$

   (E) none of the above

6. The inequality $16 - |x| \geq |3x|$ has solution

   (A) $-16 \leq x \leq 16$

   (B) $0 \leq x \leq 16$

   (C) $0 \leq x \leq 4$

   (D) $-4 \leq x \leq 4$

   (E) none of the above

7. A geometric progression P has first term 4 and sum to infinity 12. The sum to infinity of the new geometric progression obtained by squaring every term in P is

   (A) $\frac{9}{5}$

   (B) $\frac{18}{5}$

   (C) $\frac{36}{5}$

   (D) $\frac{81}{5}$

   (E) none of the above

8. The number of ways to choose a pair of distinct numbers a and b from the set $\{31, 32, \ldots, 79\}$ such that $\left|a - b\right| \leq 3$ is

   (A) 141

   (B) 144

   (C) 147

   (D) 150

   (E) none of the above

9. The area of the largest rectangle that can be inscribed in a circle of radius 16 cm is

   (A) $128\,\text{cm}^2$

   (B) $256\,\text{cm}^2$

   (C) $512\,\text{cm}^2$

   (D) $1024\,\text{cm}^2$

   (E) none of the above

10. The maximum value of the function $f(x) = (5\sin x - 3)^2 - 2$ is

    (A) 2

    (B) 7

    (C) 23

    (D) 62

    (E) none of the above

11. The function $f(x) = 2ax + b$ is such that $f(\frac{3}{2}) = 5$ and $f^{-1}(4) = \frac{1}{2}$. The value of a is

    (A) -1

    (B) $-\frac{1}{2}$

    (C) $\frac{1}{2}$

    (D) 1

    (E) none of the above

12. The vector $\begin{pmatrix} \frac{3}{5} \\ v \end{pmatrix}$ is perpendicular to the vector $\begin{pmatrix} w \\ \frac{4}{5} \end{pmatrix}$. The value of $\dfrac{v+w}{v+2w}$ is

    (A) $-\frac{1}{2}$

    (B) $-\frac{1}{5}$

    (C) $\frac{1}{5}$

    (D) $\frac{2}{5}$

    (E) none of the above

13. The derivative of $\dfrac{\ln(7-x)}{e^{7x}}$ with respect to x is

(A) $e^{-7x}\left(\dfrac{1}{x-7} - \ln(7-x)\right)$

(B) $e^{-7x}\left(\dfrac{7}{x-7} - 7\ln(7-x)\right)$

(C) $e^{-7x}\left(\dfrac{1}{x-7} + \ln(7-x)\right)$

(D) $e^{-7x}\left(\dfrac{1}{x-7} + 7\ln(7-x)\right)$

(E) none of the above

14. Two fair dice are thrown and the score on each die is noted. The probability that the scores are consecutive numbers is

(A) $\frac{3}{36}$

(B) $\frac{4}{36}$

(C) $\frac{5}{36}$

(D) $\frac{10}{36}$

(E) none of the above

15. The points P, Q and R have coordinates $(1, 4)$, (a, b) and $(4, 13)$ respectively. Suppose P, Q and R are collinear. Then the value of $\dfrac{b-4}{a-1}$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) 3

(D) 4

(E) none of the above

16. Suppose $-9 \leq x \leq 7$ and $-6 \leq y \leq 8$. Then the largest value of $y^2 + (x-2)^2$ is

(A) 81

(B) 111

(C) 145

(D) 154

(E) none of the above

17. The number of ways in which the letters of the word *ISOSCELES* can be arranged so that the last letter is not a vowel is

(A) 16800

(B) 20160

(C) 25200

(D) 30240

(E) none of the above

18. Which of the following is the result of completing the square of the expression $-20x^2 + 6x - 1$?

(A) $-20\left(x - \frac{3}{20}\right)^2 - \frac{14}{5}$

(B) $-20\left(x - \frac{3}{20}\right)^2 - \frac{29}{20}$

(C) $-20\left(x - \frac{3}{20}\right)^2 - \frac{11}{20}$

(D) $-20\left(x - \frac{3}{20}\right)^2 + \frac{4}{5}$

(E) none of the above

19. The equation of a curve is $y = 8x^3 - 12x^2 + 1$. The value of c for which the line $y = c + \frac{2}{3}x$ is a normal to the curve is

(A) -4

(B) -2

(C) 2

(D) 4

(E) none of the above

20. Which option corresponds to the partial fraction decomposition of the rational function $\frac{-13}{150x^2 - 25x - 6}$?

(A) $\frac{2}{10x - 3} + \frac{3}{15x + 2}$

(B) $\frac{2}{10x - 3} - \frac{3}{15x + 2}$

(C) $-\frac{2}{10x - 3} + \frac{3}{15x + 2}$

(D) $-\frac{2}{10x - 3} - \frac{3}{15x + 2}$

(E) none of the above

Answer FIVE (5) questions in this section. Each question carries 12 marks.

21(a). Given that $2x^3 - x^2 - 2x + 3 = (Ax + B)(x - 1)(x + 2) + C(x - 1) + D$ for all values of x, find the values of A, B, C and D. Hence, deduce the remainder when $2x^3 - x^2 - 2x + 3$ is divided by $x^2 + x - 2$. [6 Marks]

21(b). Factorise the expression $x^3 - 7x - 6$ completely. Hence, or otherwise, solve the equation

$$(y + 3)^3 + 3(y + 3)^2 - 4y = 24.$$ [6 Marks]

22(a). An arithmetic progression has 15 terms and the last term is 44. The sum of the last 10 terms is 305. Find the first term and the common difference of the arithmetic progression. [6 Marks]

22(b). The first term of a geometric progression is 3 and the common ratio is $\frac{4}{3}$. Find the greatest number of terms which can be added for their sum to be less than 200. [6 Marks]

23(a). The coefficient of x^3 in the expansion of $(1 + x)^{2n}$ and the coefficient of x^2 in the expansion of $(1 + 6x)^n$ are equal. Given that n is a positive integer and $n > 1$, find the value of n. [6 Marks]

23(b). Given that the term which is independent of x in the expansion of $(x^2 + \frac{k}{x})^6$ is 240, calculate the possible values of k. [6 Marks]

24. Find all the angles between 0° and 360° inclusive which satisfy the equation

(a) $\tan(2x - 30^\circ) = \cot 30^\circ$. [4 Marks]

(b) $3 \sin y + 2 \cos 2y = 1$. [4 Marks]

(c) $2 \cot z - \dfrac{3}{\sec z} = 0$ [4 Marks]

25(a). If one root of the equation $27x^2 + 12x + k = 0$ is three times the other, find the roots and the value of k. [6 Marks]

25(b). Solve the simultaneous equations

$$\log_2 2x + \log_4 (y+1)^2 = 3$$

$$\log_3 (x+1) = 1.$$ [6 Marks]

26(a). The line $x + y = 6$ intersects the curve $\dfrac{1}{x-1} = \dfrac{3}{y} + \dfrac{1}{4}$ at the points A and B.

(i) Find the coordinates of A and B. [4 Marks]

(ii) Given that the perpendicular bisector of AB meets the x-axis at C, find the coordinates of C. [4 Marks]

26(b). Find the range of values of p for which the expression

$$(1-2p)x^2 + 8px - (2+8p)$$

is always positive for all real values of x. [4 Marks]

END OF PAPER