Please read the follow instructions carefully.

1. This examination has TWO (2) sections – A and B, and comprises NINE (9) printed pages.

2. Attempt all sections.

3. Answer all questions in section A. Indicate your answers on the answer paper provided. Each question carries 2 marks. Marks will not be deducted for wrong answers.

4. Answer not more than FIVE (5) questions from section B. Write your answers on the answer paper provided. Begin each question on a fresh sheet of paper. Write the question number clearly. Each question carries 12 marks.

5. A non-programmable scientific calculator may be used. However, candidates should lay out systematically the various steps in the calculation.

6. At the end of the examination, attach the cover paper on top of your answer script. Complete the information required on the cover page and tie the papers together with the string provided.

7. Do not take any paper, including the question paper and unused answer paper, out of the examination hall.
Section A  (40 Marks)

Answer all questions in this section.

1. The minimum value of the function

   \[ f(x) = (x - 2006)(x - 2007)(x - 2008)(x - 2009) \]

   is

   (A) −1
   (B) 1
   (C) \( \frac{1}{4} \)(2006)(2007)(2008)(2009)
   (E) none of the above

2. There are 3 women and 5 men who will split up into two 4-person teams. The number of ways in which this can be done is

   (A) 15
   (B) 70
   (C) 90
   (D) 140
   (E) none of the above

3. Every day Peter either walks to school or takes a bus to school. The probability that he takes a bus to school is \( \frac{1}{4} \). If he takes a bus, the probability that he will be late is \( \frac{2}{3} \). If he walks to school, the probability that he will be late is \( \frac{1}{3} \). The probability that Peter will be on time for at least one out of two consecutive days is

   (A) \( \frac{49}{144} \)
   (B) \( \frac{70}{144} \)
   (C) \( \frac{84}{144} \)
   (D) \( \frac{119}{144} \)
   (E) none of the above
4. The expression \[ \frac{x^6 - 1}{x^2 - 1} \]
is equal to
(A) \( x^4 - 1 \)
(B) \((x^2 + x + 1)(x^2 - x + 1)\)
(C) \((x^2 + x + 1)(x^2 - x - 1)\)
(D) \((x^2 + x - 1)(x^2 - x + 1)\)
(E) none of the above

5. If \( f(x) = \frac{x^2 + 2x}{x - 1} \) and \( f^{-1}\left(\frac{3}{2}\right) = 7 \), then the value of \( w \) is equal to
(A) 2
(B) 3
(C) 4
(D) 5
(E) none of the above

6. The position vectors of the points \( A, B \) and \( C \), relative to the origin \( O \), are \( 12i + 20j \), \( mi + 4j \) and \(-8j\) respectively. If \( \overrightarrow{AC} \) is parallel to \( \overrightarrow{AB} \), the value of \( m \) is equal to
(A) -4
(B) -2
(C) 2
(D) 4
(E) none of the above
7. Suppose \( 4x^2 - 24x + 35 = A(x - B)^2 + C \). Then
(A) \( A = 4, B = -3 \) and \( C = -1 \)
(B) \( A = 4, B = -3 \) and \( C = 1 \)
(C) \( A = 4, B = 3 \) and \( C = -1 \)
(D) \( A = 4, B = 3 \) and \( C = 1 \)
(E) none of the above

8. The minimum value of the function
\[
f(x) = (2004 \cos 2005x - 2006)^2 + 2007
\]
is
(A) 2005
(B) 2007
(C) 2011
(D) 4013
(E) none of the above

9. The derivative of \( \ln(\cos x) \) with respect to \( x \) is
(A) \( -\frac{1}{\sin x} \)
(B) \( -\frac{1}{\cos x} \)
(C) \( -\frac{1}{\tan x} \)
(D) \( -\frac{1}{\cot x} \)
(E) none of the above
10. The derivative of $\sin(e^x)$ with respect to $x$ is
   (A) $\cos(e^x)$
   (B) $-\cos(e^x)$
   (C) $e^x \cos(e^x)$
   (D) $-e^x \cos(e^x)$
   (E) none of the above

11. The equation $9x^2 - 12x + 11 = 0$ has roots $\alpha$ and $\beta$. The value of $\alpha^2 \beta + \beta^2 \alpha$ is
   (A) $-\frac{132}{9}$
   (B) $-\frac{132}{81}$
   (C) $\frac{132}{81}$
   (D) $\frac{132}{9}$
   (E) none of the above

12. The inequality $25 - |10x + 5| \geq |40x - 20|$ has solution
   (A) $[1, 2]$
   (B) $[0, 2]$
   (C) $[-1, 1]$
   (D) $[-1, 0]$
   (E) none of the above

13. The integral

$$\int \sin^2 2x \, dx$$

equals
   (A) $\frac{1}{2} \sin^3 2x + C$
   (B) $\sin^3 2x + C$
   (C) $-\frac{1}{2} \cos^3 2x + C$
   (D) $-\cos^3 2x + C$
   (E) none of the above
14. The graph of the function \( f(x) = -x^{2008} - \cos 2007x \) is symmetric about
   (A) the \( x \)-axis
   (B) the origin
   (C) the \( y \)-axis
   (D) the line \( y = x \)
   (E) none of the above

15. Suppose \( 3^x + 3^{-x} = k \). Then the value of \( 3^{2x} + 3^{-2x} \) is equal to
   (A) \( k^2 - 2 \)
   (B) \( k^2 - 1 \)
   (C) \( k^2 \)
   (D) \( k^2 + 1 \)
   (E) none of the above

16. Five cards each have a single digit written on them. The digits are 9, 9, 8, 7, 6 respectively. The number of different 4-digit numbers that can be formed by placing four of the cards side by side is
   (A) 15
   (B) 24
   (C) 36
   (D) 60
   (E) none of the above

17. The line \( \frac{1}{2}y + 18x - q = 2 \) is a tangent to the curve \( \frac{1}{2}y = 12x^2 + 7 \). The value of \( q \) is
   (A) \( -\frac{13}{4} \)
   (B) \( -\frac{7}{4} \)
   (C) \( \frac{7}{4} \)
   (D) \( \frac{13}{4} \)
   (E) none of the above
18. The integral 
\[ \int \frac{\pi}{\sqrt{\pi^2 x + \pi}} \, dx \]
equals
(A) \( \frac{2\pi}{3} \sqrt[\theta]{(\pi^2 x + \pi)^2} + C \)
(B) \( \frac{3\pi}{2} \sqrt[\theta]{(\pi^2 x + \pi)^2} + C \)
(C) \( \frac{2}{3\pi} \sqrt[\theta]{(\pi^2 x + \pi)^2} + C \)
(D) \( \frac{3}{2\pi} \sqrt[\theta]{(\pi^2 x + \pi)^2} + C \)
(E) none of the above

19. The interval on which the function \( f(x) = 27x^3 - 108x^2 + 108x + 2008 \)
is decreasing is
(A) \([\frac{2}{3}, 2]\]
(B) \([-\frac{2}{3}, 2]\]
(C) \([-2, \frac{2}{3}]\]
(D) \([-2, -\frac{2}{3}]\]
(E) none of the above

20. The number of arrangements of all the eleven letters of the word

\( M I S S I S S I P P I \)
in which all the four letters \( I \) are consecutive is equal to
(A) \( \frac{9!}{2!4!} \)
(B) \( \frac{8!}{2!4!} \)
(C) \( \frac{7!}{2!4!} \)
(D) \( \frac{6!}{2!4!} \)
(E) none of the above
Section B  (60 Marks)

Answer not more then FIVE (5) questions in this section.

21. (a) Solve the equation \(2x^3 - x^2 - 5x - 2 = 0\).

Hence find the values of \(\theta\) for \(0^\circ \leq \theta \leq 360^\circ\) such that
\[2 \sin^3 \theta - 5 \sin \theta = 3 - \cos^2 \theta.\] [6]

(b) Find the area enclosed between the curves \(y = 9x^2 + 12x + 1\) and
\(y = -7 - 18x - 9x^2\). [6]

22. (a) Solve the simultaneous equations
\[
\log_2 4(2x + 1) + \log_4 (3y + 7)^2 = 3 \\
\log_3 (4x + 3) = 1.
\] [4]

(b) Solve the following equations
(i) \(\log_4 x = \log_2 256\). [3]
(ii) \(3^{2(x-1)} - 3^x + 2 = 0\). [5]

23. (a) The equation of a curve is \(y = 9x + \frac{4}{x}\).

(i) Find the coordinates of the stationary points. [3]

(ii) Find the equation of the normal to the curve at the point where \(x = 2\). [3]

(b) An infinite geometric progression has a finite sum of value 6. Given
that the first term exceeds the second term by twice the value of
the third term, calculate

(i) the common ratio,
(ii) the first term,
(iii) the sum from the tenth term to infinity. [6]

24. Find all the angles between \(0^\circ\) and \(360^\circ\) which satisfy the equation

(a) \(\sqrt{3} \sin^2 x = \sin x \cos x\). [3]
(b) \(2 \csc^2 x = 5(1 - \cot x)\). [4]
(c) \(\cos^2 x \cos 3x + \sin x \cos x \sin 3x = 0\). [5]
25. (a) Find the value(s) of \( k \) for which the line \( y = 3kx + 2 \) is a tangent to the curve \( 9x^2 - 6xy = 1 \). [4]

(b) Find the range of values of \( k \) for which \( 9kx^2 + 24x + 10 \geq k \) for all real values of \( x \). [4]

(c) Show that the equation \( 16x^2 + 4(1-m)x + m - 3 = 0 \) has real roots for all values of \( m \). [4]

26. (a) Find the coefficient of \( x^4 \) in the expansion of \((1 - 7x)(1 - 5x)^{11}\). [6]

(b) Find the coefficient of \( x^{-5} \) in the expansion of \((x^2 - \frac{3}{x})^{20}\). [6]

END OF PAPER.