INSTRUCTIONS TO CANDIDATES

1. This examination paper has TWO (2) sections – A and B, and comprises TEN (10) printed pages.

2. Attempt all sections.

3. Answer all questions in section A. Indicate your answers on the answer paper provided. Each question carries 2 marks. Marks will not be deducted for wrong answers.

4. Answer not more than FIVE (5) questions from Section B. Write your answers on the answer paper provided. Begin each question on a fresh sheet of paper. Write the question number clearly. Each question carries 12 marks.

5. A non-programmable scientific calculator may be used. However, candidates should lay out systematically the various steps in the calculation.

6. At the end of the examination, attach the cover paper on top of your answer script. Complete the information required on the cover page and tie the papers together with the string provided. The colour of the cover paper for this examination is GREEN.

7. Do not take any paper, including the question paper and unused answer paper, out of the examination hall.
SECTION A  (40 Marks)

Answer all questions in this section. Each question carries 2 marks.

1. The values of $w$ for which the equation $(2w + 1)x^2 + 8wx + 9 = 0$ has equal roots are

(A) $-\frac{3}{8}$ and $-\frac{3}{2}$
(B) $-\frac{3}{8}$ and $\frac{3}{2}$
(C) $\frac{3}{8}$ and $-\frac{3}{2}$
(D) $\frac{3}{8}$ and $\frac{3}{2}$
(E) none of the above

2. The line $y = 2x - 6$ meet the curve $y^2 = 16x$ at the points $A$ and $B$. The length of $AB$ is

(A) $8\sqrt{5}$
(B) $12\sqrt{5}$
(C) $16\sqrt{5}$
(D) $24\sqrt{5}$
(E) none of the above

3. The range of values of $p$ for which $8 - 3x - x^2 \leq p$ for all real values of $x$ is

(A) $p \geq \frac{41}{4}$
(B) $p \leq \frac{41}{4}$
(C) $p \geq -\frac{43}{4}$
(D) $p \leq -\frac{43}{4}$
(E) none of the above
4. The expressions $x^3 - x^2 - 4x + 24$ and $x^3 - 7x + 6$ leave the same remainder when divided by $x - c$. The possible values of $c$ are

(A) $-3$ and $-6$
(B) $3$ and $-6$
(C) $-3$ and $6$
(D) $3$ and $6$
(E) none of the above

5. The derivative of $x\sqrt{x+1}$ with respect to $x$ is

(A) $\frac{2x + 1}{2\sqrt{x+1}}$
(B) $\frac{2x + 1}{\sqrt{x+1}}$
(C) $\frac{3x + 2}{\sqrt{x+1}}$
(D) $\frac{3x + 2}{2\sqrt{x+1}}$
(E) none of the above

6. The inequality $16 - |x + 2| \geq |3x + 6|$ has solution

(A) $-4 \leq x \leq 4$
(B) $-6 \leq x \leq 2$
(C) $-2 \leq x \leq 6$
(D) $-6 \leq x \leq 4$
(E) none of the above
7. The first three terms of an arithmetic progression are \( \lg\left(\frac{1}{3} x\right)\), \( \lg\left(\frac{1}{3} x + 2\right) \) and \( \lg\left(\frac{1}{3} x + 16\right) \).

The value of \( x \) is

(A) \( \frac{1}{3} \)

(B) \( \frac{1}{3} \)

(C) 1

(D) 3

(E) none of the above

8. A tennis team of 4 men and 4 women is to be selected from 6 men and 7 women.

It was decided that two particular men \( A \) and \( B \) must either be selected together or not selected at all. The number of ways in which this can be done is

(A) 210

(B) 225

(C) 245

(D) 375

(E) none of the above

9. Suppose \( 2x^3 + 3x^2 - 14x - 5 = (Px + Q)(x + 3)(x + 1) + R \) for all values of \( x \). Then

(A) \( P = 2, Q = -5 \) and \( R = 10 \)

(B) \( P = 2, Q = -5 \) and \( R = -20 \)

(C) \( P = 2, Q = 5 \) and \( R = 10 \)

(D) \( P = 2, Q = 5 \) and \( R = -20 \)

(E) none of the above
10. The maximum value of the function \( f(x) = (7 \cos x - 6)^2 - 5 \) is

(A) 5
(B) 41
(C) 164
(D) 174
(E) none of the above

11. The function \( f(x) = 8ax + b \) is such that \( f\left(\frac{3}{8}\right) = 5 \) and \( f^{-1}(4) = \frac{1}{8} \). The value of \( a \) is

(A) \(-1\)
(B) \(-\frac{1}{2}\)
(C) \(\frac{1}{2}\)
(D) 1
(E) none of the above

12. The vector \( \begin{pmatrix} 5 \\ v \end{pmatrix} \) is perpendicular to the vector \( \begin{pmatrix} w \\ 12 \end{pmatrix} \). The value of \( \frac{5v + 12w}{5v - 12w} \) is

(A) \(-\frac{119}{169}\)
(B) \(\frac{119}{169}\)
(C) \(-\frac{169}{179}\)
(D) \(\frac{169}{179}\)
(E) none of the above
13. The curve for which \( \frac{dy}{dx} = 5x + k \), where \( k \) is a constant, has a turning point at \((-2, -1)\). The value of \( k \) is

(A) 10
(B) \( \frac{2}{5} \)
(C) \( -\frac{2}{5} \)
(D) -10
(E) none of the above

14. Two fair dice are thrown and the score on each die is noted. The probability that the sum of the scores is greater than 4 is

(A) \( \frac{2}{3} \)
(B) \( \frac{3}{4} \)
(C) \( \frac{5}{6} \)
(D) \( \frac{11}{12} \)
(E) none of the above

15. The points \( P, Q \) and \( R \) have coordinates \((2, 7), (a, b)\) and \((5, 16)\) respectively. Suppose \( P, Q \) and \( R \) are collinear. Then the value of \( \frac{b - 4}{a - 1} \) is

(A) \( \frac{1}{4} \)
(B) \( \frac{1}{3} \)
(C) 3
(D) 4
(E) none of the above
16. Suppose \(-9 \leq x \leq 7\) and \(-6 \leq y \leq 8\). Then the smallest value of \(y^2 + y + x^2 + 3x + \frac{5}{2}\) is

(A) \(-\frac{71}{4}\)
(B) \(-\frac{5}{2}\)
(C) 0
(D) \(\frac{5}{2}\)
(E) none of the above

17. The number of ways in which the letters of the word \textit{INCLUDE} can be arranged so that each arrangement begins with a consonant and ends with a vowel is

(A) 144
(B) 576
(C) 720
(D) 1440
(E) none of the above

18. Which of the following is the result of completing the square of the expression \(-3x^2 + 8x - 5\)?

(A) \(-3\left(x - \frac{4}{3}\right)^2 - \frac{1}{3}\)
(B) \(-3\left(x - \frac{4}{3}\right)^2 + \frac{1}{3}\)
(C) \(-3\left(x - \frac{4}{3}\right)^2 - \frac{31}{3}\)
(D) \(-3\left(x - \frac{4}{3}\right)^2 - \frac{16}{3}\)
(E) none of the above
19. The equation of a curve is \( y = 6x + \frac{4}{x} \). The equation of the normal to the curve at the point \( x = 1 \) is

(A) \( 2y = -x - 21 \)

(B) \( 2y = -x + 21 \)

(C) \( 2y = 2x - 21 \)

(D) \( 2y = 2x + 21 \)

(E) none of the above

20. Which option corresponds to the partial fraction decomposition of the rational function \( \frac{-17}{12x^2 - x - 6} \)?

(A) \( \frac{-4}{4x - 3} + \frac{3}{3x + 2} \)

(B) \( \frac{4}{4x - 3} + \frac{3}{3x + 2} \)

(C) \( \frac{-4}{4x - 3} - \frac{3}{3x + 2} \)

(D) \( \frac{4}{4x - 3} - \frac{3}{3x + 2} \)

(E) none of the above
Answer FIVE (5) questions in this section. Each question carries 12 marks.

21(a). Differentiate $2x\sqrt{12x + 1}$ with respect to $x$. Hence, evaluate $\int_{0}^{1} \frac{108x + 6}{\sqrt{12x + 1}} \, dx$. [6 Marks]

21(b). Find the area enclosed between the curves $y = 4x^2 - 16x + 13$ and $y = -7 + 12x - 4x^2$. [6 Marks]

22(a). Suppose the expansion of $(1 + px)^8(1 - 11x)^2$ in ascending powers of $x$ is

$$1 + 18x + qx^2 + \ldots.$$ 

Find the value of $p$ and of $q$. [6 Marks]

22(b). Find the coefficient of $x$ in the expansion of $(4x - 3x^2)^{16}$. [6 Marks]

23(a). Factorize completely the expression $4x^3 - 13x - 6$ and hence solve the equation $2(2x^2 - \frac{3}{4}) = 13$. [6 Marks]

23(b). Given that $3x^2 - 7x + 4 = A(x - 2)^2 + B(x - 2) + C$ for all values of $x$, find the values of $A$, $B$ and $C$. [6 Marks]

24. Find all the angles between $0^\circ$ and $360^\circ$ inclusive which satisfy the equation

(a) $\tan 2x = \cot 40^\circ$. [4 Marks]

(b) $4 \sin y + \tan y = 0$. [4 Marks]

(c) $2 \sec z + \tan^2 z = 7$. [4 Marks]
25(a). Find the value(s) of $p$ for which the line $y = p + x$ is a tangent to the curve $xy + x^2 = -2$. [6 Marks]

25(b). Solve the simultaneous equations

\[
(5^x)(25^{2y}) = 1 \\
(3^{5x})(9^y) = \frac{1}{9}.
\] [6 Marks]

26(a). A curve is given parametrically by the equations

\[
x = \frac{6}{(1+t)^2} \quad \text{and} \quad y = \frac{3t}{1+t}, \quad \text{where} \quad t \neq -1.
\]

Let $P$ be the point on the curve with $t = 0$

(i) Find the points of intersection of the curve with the line $y - x = 0$. [4 Marks]

(ii) Find the equation of the normal at the point $P$. [4 Marks]

26(b). A curve is given parametrically by the equations

\[
x = \cos \theta - 2\sin \theta \quad \text{and} \quad y = 2\cos \theta + \sin \theta.
\]

Find the cartesian equation of the curve. [4 Marks]

END OF PAPER