Stable Marriage Problem: The Mean Performance Analysis of the Fundamental Algorithm and Algorithmic Analysis of Several Variants

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Abstract

The topics being investigated in this project are centered around the classic Stable Marriage Problem, which can be divided into two aspects, mathematical analysis in the mean performance of the fundamental algorithm and algorithmic method in tackling problems regarding several variants.

The definition of the fundamental problem can be summarized as follows: Given $n$ men and $n$ women and their preference lists (in the form of ranking), find a stable matching such that there does not exist a pair $(m,w)$, where $m$ stands for a man and $w$ stands for a woman, such that $m$ and $w$ are not paired up but they prefer each other to their respective partners.

One of the major variants that are being investigated is Stable Marriage Problem with Ties and Partially ordered lists. The problem involves the instance where the preference lists do not only admit strict order but also admit partial order: a man (woman) can be indifferent between two women (men) and two women (men) can be incomparable to a man (woman). Note the indifference relationship is transitive while the comparability relationship is not transitive.
The main focuses and contributions of this project are:

a) investigating an open problem posed by Donald Knuth (The Third Research Problem in Lecture 7);

b) devising a $O(n^2)$ algorithm to determine a stable solution given forced pairs;

c) Finding an alternative method of solving stable marriage problem with ties and incomplete list under strong stability assumption in polynomial time;

d) Showing the NP-Completeness of problem instance with partially ordered complete (or incomplete) preference lists.