Stern-Gerlach Interferometer with Realistic Magnetic Field

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Abstract

In this report, we investigate the spin dynamics of a neutron beam in a Stern-Gerlach experiment. In contrast to the simple constant gradient magnetic field assumed in most literatures which violates Maxwell Equations, we work with a model of magnetic field which satisfies Maxwell Equations. The spin dynamics is investigated by solving the Schrödinger equation using Fast Fourier Transform method. The spin coherence of the neutron beam is found to exhibit the humpty-dumpty behaviour[1, 2, 3] even though there is no fluctuation to the magnetic field nor any environmental noise. The main cause of the spin decoherence is identified as the inhomogeneity of the magnetic field in the x and y components. In addition, the nonlinear terms in the z component of the magnetic field, z^n, $n > 1$ also contribute to the loss of the spin coherence. Although only magnetic field model considered is very specific, the cause of the loss of spin coherence identified using this model is a common features of any realistic magnetic to be used in a SGA experiment. Therefore the humpty-dumpty nature of the spin coherence exist even without any fluctuation to the magnetic field and is inherent to the SGA experiment itself.

INTRODUCTION

Stern-Gerlach Apparatus (SGA) is by far one of the many important experiments done in the history of Physics. The outcome of the experiment done in 1922 by Otto Stern and Walther Gerlach clearly showed the quantization nature of the physical property called spin of spin-$\frac{1}{2}$ particles. It is also one of the earliest experimental evidences showing the quantization of intrinsic angular momentum or spin. The basic ideas behind SGA is to show the spacial quantisation of a beam of spin-$\frac{1}{2}$ particles after their quantized spin interacts with an inhomogeneous magnetic field.

There are many recent proposals [4, 5] which could be realized with an efficient SGA interferometer. However, most of SGA treatment assumes a simplified form of magnetic field which does not satisfy Maxwell Equations. The unavoidable inhomogeneity field in the x and y component has been argued to have significant effect in the spin coherence of SGA. In [1], the authors has shown that a precision of at least one part in 10^5 in the magnetic field's inhomogeneity is required to maintain spin coherence. In this project, we go further by investigating the possibility of performing interferometry with SGA.

MODEL AND METHOD

We first come out with a form of magnetic field which satisfy the Maxwell Equations. Our approach is to exploit the symmetry in the four Maxwell Equations when the region considered has no current and charge density, $\vec{j} = 0$ and $\rho = 0$. We could therefore use magnetic scalar potential to formulate our magnetic field model based on hypothetical magnetic point charges.

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The magnetic field obtained drops in the order $\frac{1}{y^5}$ and this is good enough so that we do not need to inject neutron beam from too far away.

Then we seek to numerically solve the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} = -\frac{\hbar^2}{2M}\nabla^2\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} - \mu_0\vec{\sigma}\cdot\vec{B}(\vec{R})\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}, \tag{1}$$

where the initial state of the system is in the spin $+x$ state and carries a Gaussian wave packet, as given by the total wave function

$$\psi(\vec{r},0) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}\frac{(2\pi)^{-3/4}}{(\delta x)^{3/2}}\exp\left(-\left(\frac{|\vec{r}|}{2\delta x}\right)^2\right)\exp\left(\frac{ip_y y}{h}\right). \tag{2}$$

It is helpful to transform the Schrödinger equation such as to remove some of the fastly changing features in the equation. Under such transformation,

$$\psi(\vec{r},t) \rightarrow U(t)\psi(\vec{r},t), \tag{3}$$

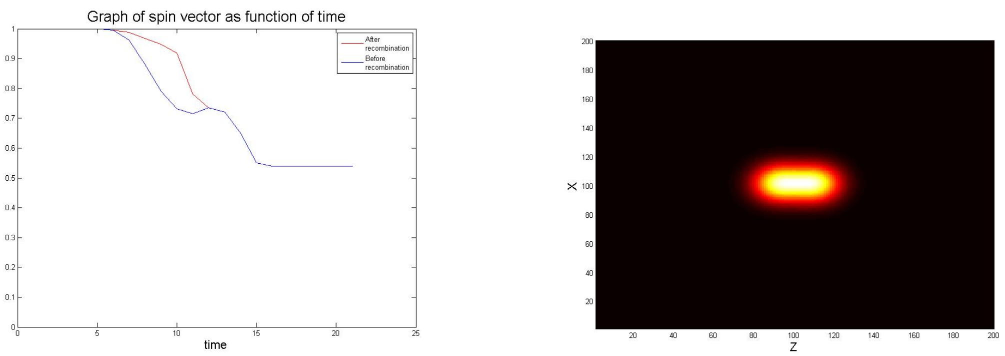
where

$$U(t) = \exp\left(\frac{ip_y y}{\hbar}\right)\exp\left(-\frac{it}{\hbar}\frac{p^2}{2M}\right)\exp\left(-\frac{i}{\hbar}\int dt\,\frac{p_z(t)^2}{2M}\right)\exp\left(-\frac{i}{\hbar}\mu_0\int dt\,\vec{B}'(t)\cdot\vec{\sigma}\right). \tag{4}$$

The Schrödinger equation becomes

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} = & \Big[-\frac{\hbar^2}{2M}\nabla^2 - i\hbar\frac{p}{M}\frac{\partial}{\partial y} - i\hbar\frac{\mu_0\sigma_z}{M}\int_0^t dt\,B_1(t)\frac{\partial}{\partial z} \\ & -\exp\left(\frac{i}{\hbar}\int dt\,\vec{B}'(t)\cdot\vec{\sigma}\right)\mu_0\vec{\sigma}\cdot(\vec{B}(\vec{R})-\vec{B}'(t))\exp\left(-\frac{i}{\hbar}\int dt\,\vec{B}'(t)\cdot\vec{\sigma}\right)\Big] \\ & \begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}. \end{aligned} \tag{5}$$

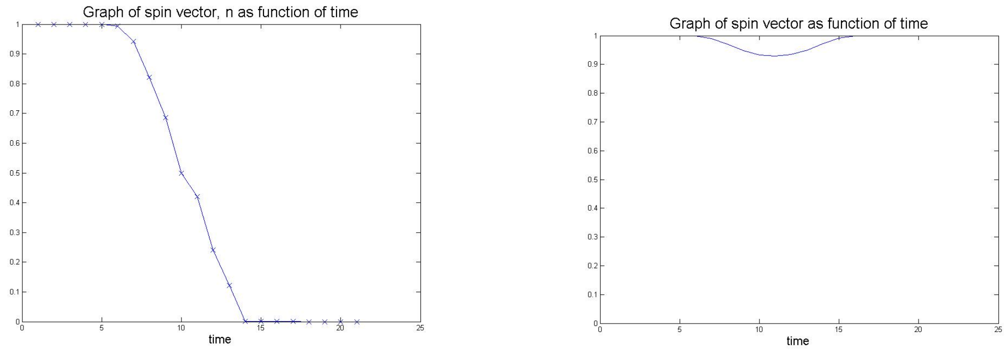
RESULTS



(a) Spin vector dynamics. (b) Wave profile.

Figure 1: Dynamics of the spin vector and wave profile of the 2 beams when they are maximally separated. $B = 1.2$.

The spin vector dynamics of the neutron beam is investigated by varying the magnetic field strength. Figure 1 shows the behaviour for $B = 0.96$. The spin vector is recovered partially



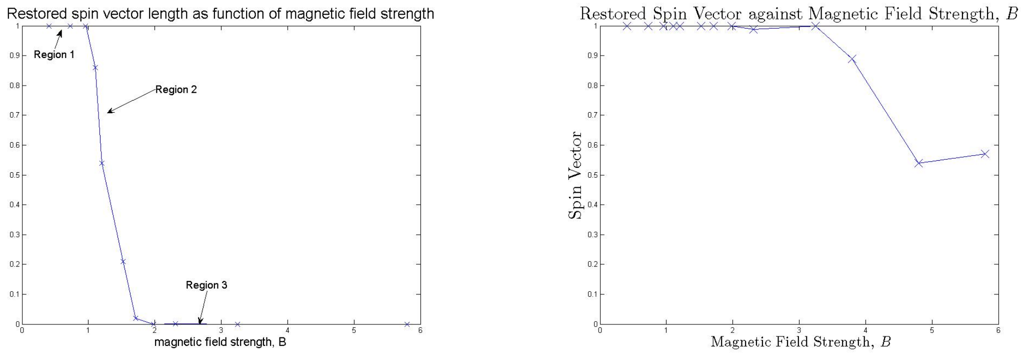
(a) $B = 5.8$.

(b) $B = 0.96$.

Figure 2: Dynamics of spin vector for $B = 5.8$ and $B = 0.96$.

after the two wave packets are recombined. The red curve in Figure 1a is the spin vector when they are recombined artificially at each instant so as to remove the effect caused by the spatial separation of the two beams, which is recoverable in principle.

Figures 2a and 2b show the behaviour when the magnetic field is $B = 5.8$ and $B = 0.96$ respectively. For these two cases, the red curves coincide with the original. Now if we plot the spin vector recovered at the end of SGA against the magnetic field strength, we obtain the graph as shown in Figure 3a.



(a) Complete magnetic field.

(b) Partial magnetic field.

Figure 3: Dynamics of spin vector for $B = 5.8$ and $B = 0.96$.

There are three regions in Figure 3a. When $B < 1$, complete restoration of the spin vector is achieved. When $1 < B < 2$, partial restoration of the spin vector length is possible. However, at this range, the final spin vector restored drops quickly. When $B > 2$, the spin vector is lost completely.

To understand the cause of the loss of spin vector better, the experiment is repeated by dropping the x and y component of the realistic magnetic field. Under such circumstances, the behaviour of the spin vector is as shown in Figure 3b.

DISCUSSION

Our results show that even under the idealized condition where there is no fluctuation to the magnetic field and environmental interaction, it is still difficult to restore the spin coherence. Furthermore, in the case where spin coherence is able to be restored either partially

or completely, the maximum separation of the two wave packets are generally very small, on the order of 1mm. This makes SGA very difficult for any practical interferometer experiment.

Furthermore, from the results, we are able to identify the cause of the loss of spin coherence to be the inhomogeneity of the x and y component of the magnetic field. Furthermore, the higher order terms of z, z^n where $n > 1$ in the z component of the magnetic field is also identified to cause the loss of spin coherence. Even though we have assumed a particular model of megnetic field, the features causing the loss of spin coherence is a common features in any SGA model. In short, it is not easy to have interferometry by using SGA even though we are able to perform the experiment free from any inevitable fluctuations and noise from the environment. This further shows that the humpty-dumpty nature of the spin coherence is inherent in the SGA experiment itself.

CONCLUSION

By simulating a neutron beam passing through a particular realistic magnetic field, we can investigate the dynamics of the neutron beam and its spin by solving the Schrödinger equation . Our numerical results suggest that even though we have perfect experimental condition where there is no fluctuation to the magnetic field and no enviromental interaction, the spin coherence is still humpty-dumpty in nature. In particular, if the neutron beam is split for more than 2 times the initial spread of the gaussian wave packet, the spin coherence is lost permanently. By comparing the results with a hypothetical form of magnetic field without the x and y component, we can identify some of the features of the realistic magnetic field which is causing the loss of spin coherence. These features identified are common to any practical magnetic field in any SGA experiment. Therefore, the humpty-dumpty nature of the spin coherence is an inherent feature of any SGA experiment.

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