Review on Causal Dynamical Triangulation as a candidate of the quantum
theory of gravity

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In this undergraduate research opportunities program in science (UROPS), we study one of the most successful candidates of the theory of quantum gravity, the Causal Dynamical Triangulation (CDT). The project was motivated by the publication [1] by Ambjørn, Jurkiewicz and Loll in the success of CDT producing the correct dimension of our universe in 2004. It marked the first time that a candidate of the quantum theory of gravity had successfully delivered the correct dimension from first principles.

1 Introduction

In the current context of our understanding of the universe, we have quantum physics which explains our microscopic regime and classical theories which describe our macroscopic world, demarcating the two “extreme range” of observation. Gravity in our observable or observed dimensions of the universe are well described by the general theory of relativity (GR), it shows that both matter (mass) and space-time structures are correlated and co-exist with one and other. However in quantum physics, the arena for the dynamics of the system is usually formulated on a flat, four dimensional Minkowski space. Obviously, there is a conflict of background here as deem by GR and quantum physics. Popular models like the string theory and loop quantum gravity (LQG) are painstakingly developed but none have produced convincing results yet. However a life line was thrown in 2004 when causal dynamical triangulation (CDT) managed to approximate the correct dimension of our universe. With this convincing candidate, we look into the concepts and methodologies and try to interpret its capabilities and limitations.

2 Causal Dynamical Triangulation

In causal dynamical triangulation, the main framework mainly consists of two tools. First, the Feynman’s path integral with each possible evolution of the space-time structures as the configurations of virtual paths and the second tool as Regge calculus. In figure 1, we listed the main methodology of CDT starting from path integral.
3 Gravitation quantum path integral

The gravitation quantum path integral used in CDT is defined as the partition function of quantum gravity as a formal integral over all space-time structures weighted by the complex exponent of the classical Einstein-Hilbert action. It is given as

\[ Z \left( g(0), g(t); t \right) := \int_{\mathcal{P}} D(g) \exp \left( iS_{\text{Einstein-Hilbert}}(g) \right) \] (1)

with the classical Einstein Hilbert action as

\[ S_{\text{Einstein-Hilbert}}(g) := \frac{1}{G_N} \int d^4x \sqrt{\det(g)} \left( R - 2\Lambda \right) \] (2)

Where \( g \) is the space-time metric dependent on time \( t \), \( G_N \) is the Newton gravitation constant, \( R \) the Ricci scalar and \( \Lambda \) is the cosmological constant. The importance of dependence on \( \Lambda \) is not obvious until when CDT tries to generate a de Sitter universe\[6\], \[7\]. By invoking Regge calculus, one introduces regularization to the class of diffeomorphism space-time structures. So instead of integrating over all space-time geometries, one do a summation of all causal triangulations. This gives

\[ \int_{\mathcal{P}} D(g) \exp \left( iS_{\text{Einstein-Hilbert}}(g) \right) \rightarrow Z \left( G_N, \Lambda \right) := \sum_{g \in \mathcal{P}} C_g \exp \left( iS_{\text{Regge}}(g) \right) \] (3)

The key idea next is to employ CDT Wick rotation to select out a class or set of triangulations \( T \) from triangulation space \( \mathcal{T} \), where the triangulation space consists of all possible triangulations of all smooth space-time structures. By invoking the CDT’s Wick
rotation, the equation (3) is transformed into a problem of statistical physics, namely, the evaluation of a partition function

\[ Z(G_N, \Lambda) := \sum_{T \in T} \frac{1}{C_g} \exp (-S_{\text{Regge}}(T_{\text{Euclidean}})) \]  

(4)

4 Monte Carlo Simulation guide of 3-D universe

For simplicity reasons, we study a publication by Zhang [5] in his numerical method for a 3 dimension universe. From the previous sections, the path integral is now well-defined and computable. The integration space or summation is to be over all Euclidean-Regge triangulations (each corresponds to the constructed Lorentzian triangulation) hence equation (3) can be written as

\[ Z(k_0, k_3) = \sum_{T} \frac{1}{C_g} \exp (k_0 N_0 - k_3 N_3) \]  

(5)

For a 3 dimensional problem, our building blocks or simplices are limited to just 2 configurations but 3 unique tetrahedra. They are \((3,1), (1,3)\) and \((2,2)\) tetrahedra. Basically, these moves will be the Monte Carlo moves. One now has defined the Monte Carlo moves and can proceed to evaluate the summation in expression (11). The expectation value of an observable with \(N\) measurements as

\[ \langle O \rangle := \frac{\sum O(T) \exp (k_0 N_0 - k_3 N_3)}{Z(k_0, k_3)} \approx \frac{1}{N} \sum_{i} O(C_i) \]  

(6)

What is left now is to run the Monte Carlo moves to explore the phase space of the system. Zhang [5] used two test configurations to probe the phase change in the system, the two test space-time configurations are:

\[ \text{Triangulation 1} := \left\{ \frac{N_{(2,2)}}{N_3} \right\}_{T=8,V=8000} \]  

(7)

\[ \text{Triangulation 2} := \left\{ \frac{N_{(2,2)}}{N_3} \right\}_{T=32,V=32000} \]  

(8)

With these two test configurations, the critical point is observed to be around \(k_0 = 3.3\) and from this value, the before and after phase changes were plotted for spatial volume \(N_2\) against time as shown in figure 2.
From figure 5, one can see that the left plot for the phase space $k_0 < 3.3$ shows a well behaved geometry. What is important here is that the successive spatial slices are strongly coupled and exhibit near classical universe structure.

5 Discussions and Conclusion

As a summary, we would like to say that the Causal Dynamical Triangulation has successfully shown that it can retrieve back the observed universe at large scales. It is developed from first principles based on independent of background and non-perturbative approach. However, in its manner of progression, it employs techniques that are only computable through Monte Carlo simulations. One must be careful here because of the infinitely free parameters that one can play with. It is like introducing more and more degrees of freedom along the way to achieve some desired effects. Nevertheless, unless CDT can predict some observable phenomena that can be measured, this candidate can only serves as a theory to pseudo quantum theory of gravity.

References


