Simulations of One-Dimensional Electromagnetic Waves in Lorentz Media

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ABSTRACT
The purpose of the research is to investigate the propagation of one-dimensional electromagnetic waves in Lorentz media, including “Lorentz models” of negative index materials. Finite Difference Time Domain (FDTD) simulations were performed using Matlab. The behaviours of phase and group propagation in the media, and the penetration of an electromagnetic wave into slabs of material were observed and analyzed, and compared with theory.

Method to create electromagnetic simulations
A Lorentizian model with relative permittivity is given by

\[ \varepsilon_r = 1 + \frac{\omega_p^2}{(\omega_p^2 - \omega^2) + 2i\gamma\omega} \]  \hspace{1cm} (1)

where \( \omega_p^2 \), the square of plasma frequency, is given by \( \frac{Ne^2}{mc_0} \) and \( \varepsilon_r \) is the relative permittivity of the Lorentz medium, and the permittivity is given by

\[ \varepsilon = \varepsilon_0 \varepsilon_r \]  \hspace{1cm} (2)

In the first part of our study, we assume the permeability, \( \mu_0 \). However in the second part of our study the permeability is also assumed to be Lorentzian in order to provide a platform for us to discuss electromagnetic wave propagations in negative refractive index materials. The Lorentzian permeability is assumed to be,

\[ \mu(\omega) = \mu_0 \left\{1 + \frac{\omega_p^2}{[(\omega_p^2 - \omega^2) + 2i\gamma\omega]} \right\} \]  \hspace{1cm} (3)

The properties of an electromagnetic wave are given by its electric field and magnetic field. In a medium, we have the following constitutive relations:

\[ \overrightarrow{D} = \varepsilon \overrightarrow{E} \]  \hspace{1cm} (4)

\[ \overrightarrow{B} = \mu \overrightarrow{H} \]  \hspace{1cm} (5)

In the absence of free charge and free current, we have two Maxwell’s equations in its simplest form

\[ \nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} \]  \hspace{1cm} (6)

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\[ \bar{\nabla} \otimes \bar{H} = \frac{\partial \bar{D}}{\partial t} \] (7)

To create electromagnetic simulation models of one dimensional monochromatic electromagnetic waves, the Finite Difference Time Domain (FDTD) simulations were performed using Matlab. For the case of non-magnetic materials, \( \mu = \mu_0 \), Eq. (7) enables us to compute the H field values at time \( t = (n+1)\Delta t \) at grid point values \( z = (m+1/2)\Delta z, m=1, 2, 3 \ldots M \), with the values of E fields at time \( t = (n+1/2)\Delta t \) at grid point values \( z = m\Delta z \) assumed known (Sullivan, 2000):

\[ H_{m+\frac{1}{2}}^{n+1} = H_{m+\frac{1}{2}}^{n} - \left( \frac{\Delta t}{\mu_0 \Delta z} \right) (E_{m+\frac{1}{2}}^{n+1} - E_m^{n+\frac{1}{2}}) \]

Similarly the prediction for the electric displacement \( D \) is given by Eq. (6)

\[ D_{m+\frac{1}{2}}^{n+1} = D_{m+\frac{1}{2}}^{n} - \left( \frac{\Delta t}{\Delta z} \right) (H_{m+\frac{1}{2}}^{n+1} - H_{m-\frac{1}{2}}^{n+1}) \]

To calculate the electric field from the displacement, we need to explicitly predict the polarization field as well

\[ P_{m+\frac{1}{2}}^{n+1} = P_{m+\frac{1}{2}}^{n} \left( \frac{2 - \omega_0^2 \Delta t^2}{1 + \gamma \Delta t} \right) + P_{m+\frac{1}{2}}^{n} \left( \frac{\gamma \Delta t - 1}{\gamma \Delta t + 1} \right) + E_m^{n+\frac{1}{2}} \left( \frac{\varepsilon \omega_0^2 \Delta t^2}{1 + \gamma \Delta t} \right) \]

When \( D \) and \( P \) at time \( t=(n+1/2)\Delta t \) are computed, \( E \) at time \( t=(n+1/2)\Delta t \) is calculated as

\[ E_{m+\frac{1}{2}}^{n+1} = \frac{D_{m+\frac{1}{2}}^{n+1} - P_{m+\frac{1}{2}}^{n+1}}{\varepsilon_0} \]

With the above equations, we will be able to simulate the future evolution of an electromagnetic wave given its field variables at the relevant grid points and previous time steps.

**Results and Discussion**

We tested whether the FDTD model is accurate and it was found to be so. This is obtained by checking whether the simulated group and phase velocity obtained from FDTD method agrees with the theoretical values. It was found that the simulated group and phase velocity were close to theoretical values.

![Fig. 1. Phase velocity as function of real values of k for Lorentz Medium.](image-url)
Fig. 2. Group velocity (cg) as function of real values of k for Lorentz medium

Also, when a pulse is generated to propagate in materials of both positive and both negative permeability and permittivity, observations agree with theory.

When a pulse was sent through a medium with positive Lorentzian permittivity and permeability of free space, the reflected and transmitted coefficient, \( R_E \) and \( T_E \) of the electromagnetic waves were found and were found to be close to the ratio 1. This agrees with the theory which is given by

\[
1 + R_E = T_E
\]

Fig. 3. Plot of computed \( T_E/(1+R_E) \) versus \( \omega \) represented by ‘dashed-dotted’ line.

When a wave packet was sent into a medium with positive Lorentzian permittivity and positive Lorentzian permeability, spreading of wave was observed and this agrees with theory since the plotted group velocity generally decreases for large values of k.
When a wave packet was sent into a medium with negative refractive index as assumed by the Lorentzian model, waves were seen being propagated to the left even though there is a growth in the number of peaks in the medium. This agrees with theory since the Poynting vector and wave number vector is in the opposite direction when wave is propagated in negative refractive index materials.

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Reference