Applications of Production Decision Models under Risk

A Decision Analysis Approach with Incorporation of Behavioral Utilities

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ABSTRACT

A conceptual model examines the competitive firm’s optimal input and hedging decisions under price and output uncertainty. Behavioral utility functions are incorporated to represent the firm’s irrational organizational behaviors. The model is then solved in a numerical example using decision analysis methods. It is shown to be capable to solve for the optimal decisions under different market conditions and characteristics of the firm. Subsequent sensitivity analyses suggest that optimal level of hedging is heavily influenced by the futures price and the firm’s risk preferences.

1. INTRODUCTION

The uncertain, indeterminable and responsive nature of economic activities leads to the common existence of risk. Apart from the resource constraints such as capital, labor and productivity, a producer's output decision is also associated with the risk he is facing and his attitude towards it. Given a basket of alternatives, risk-savvy producers in the market would always tend to seek an ideal trade-off point between risk and expected return. In this paper, we will look into the implications of real-world organizational behaviors in the production decision problems under both price and output uncertainty.

This paper is organized as followed. In section 2, we present a literature review of existing production decision models. A typical decision model is introduced in section 3 under expected utility maximization. The model is then solved in some numerical examples using decision analysis in section 4 where different behavioral utility functions will also be discussed. Section 5 takes a closer look at the dependency of production decisions on market conditions and risk preferences by performing several sensitivity analyses. Finally, section 6 concludes with some suggestions to further study.

2. LITERATURE REVIEW

The majority of existing models of production decision under uncertainty are based on the expected utility approach. On the optimal level of production, the firm maximizes the expectation of its utility, which is a function of wealth (or profit). The utility function is
normally assumed to be increasing, twice-differentiable and concave to represent the rise-averse preference of the firm.

2.1 Production Decision under Uncertainty

In his now well-cited 1971 paper, Sandmo proposed a one-period, single commodity model to study the competitive firm's output decision under price uncertainty and drew the conclusion that under price uncertainty, output tends to be smaller than the certainty output. He also analyzed the effects of change in the expected value and uncertainty of the price on the optimal output of the competitive firm and demonstrated that the firm's response to an increase in tax rate is dependent on its relative risk aversion.

The subsequent literature following Sandmo attempted to alter one or few assumptions made in his model. Batra & Ullah (1974) studied the problem in the long-run, which was further improved by Paris (1988) who introduced both input and output price uncertainties. Batra & Ullah (1974) also proved that cost minimization is consistent with the maximizing expected utility of wealth of a firm facing riskless production but output price uncertainty. Pope and Chavas (1994) extended this consistency to the case where production is also under uncertainty.

2.2 Some Variations

The single-commodity, single-period limitation was also loosened in various ways. Chavas and Holt (1990) proposed an acreage decision model where producers are able to distribute his acreage resource (an input factor) among two types of commodities, namely corn and soybeans, also taking into consideration the wealth effect. The model was used in an application under government price support programs. Babcock (1990) provided a two-period model on agricultural production where unused marketing quota (an input factor) can be carried on to the next period. Lence & Hayes (1998) constructed a model in a multi-period scheme where firms are able to store its output for sale in the subsequent periods. They concluded that forward-looking, risk-averse firms will produce more than risk-neutral ones.

Another important aspect of improving the Sandmo model is to involve acquisition of information and adoption to technology. Information about price and/or technology, whose acquisition involves certain cost, can be important to the firm's production decision making. Kihlstrom (1976) was one of the earliest studies on determining the firm's demand for information, showing that increase in level of information rises expected output, and a downward-sloping demand curve for that information, which is in consistence with any other kind of normal goods. Eggert & Tveiteras (2004) studied fishers' gear choices (representing a certain type of production technology).

Saha et al. (1994) suggested that the adoption of technology, also being expensive, can lead to an increase in the firm's expected output as well as a reduction in its variance (risk). It turned out that the adoption decision is determined by the expected revenue and cost of the technology, which is in turn dependent on the amount and completeness of information available to the producer. The risk factors only influence the optimal level of adoption. Koundouri et al. (2006) performed a study on adoption of irrigation technology. In their scenario, technology was treated as a more efficient alternative to the current risky production method. Adoption of technology thus became a means of hedging against production risk.
2.3 **The Use of Financial Derivatives**

The introduction of financial derivatives has become an influential way for the suppliers to reduce their risk exposure by selling future products for a certain price. Considerable research has focused on the topic of production decision under risk with the existence of futures and forward markets. Holthausen (1979) studied the choice of competitive firm under price uncertainty in such context. With future markets, the producer has to make both production and hedging decisions. It is showed that a risk-averse producer's output decision will be the level for which marginal cost equals the forward price, regardless of their risk expectations or preferences. However, risk aversion does play a decisive role on the firm's optimal hedging levels. Grant (1985) further incorporated the possibility of both price and output uncertainty into the model of risk-averse competitive firm with access to a forward market and his results turned out to be revolutionary. In the presence of both price and output uncertainty, future markets can no longer completely remove all the risk, and the producer's decision making is dependent on many factors such as its risk preference and the correlation between revenue and price. Such problems are to be solved on a case-by-case basis.

The emergence of options provides producers with more hedging choices. Lapan, Moschini & Hanson (1991) added options market to the expected utility model. In their model, firm makes decision on the production scale, and the quantity of futures and options contracts purchased. Uncertainty exists in end-of-period price while the production function is assumed to be non-stochastic. They concluded that when futures prices and options premiums are perceived as unbiased, the optimal hedging decision only involves futures. Otherwise, options shall be used together with futures. Sakong, Hayes & Hallam (1993) further incorporated production uncertainty in this model, and showed that producers can partially offset the risk in production by hedging some of their expected output in the futures market and purchasing put options.

2.4 **Behavioral Utilities**

The assumption that the utility function appears to be increasing and concave is challenged various ways. Kahneman and Tversky (1979) studied loss aversion which takes into consideration the fact that decision making is partially influenced by current assets. Their empirical studies revealed the loss aversion function which is convex and increasing below a certain reference point. Pennings and Smidts (2003) looked into the shape of utility and organizational behavior. More is discussed in Tversky’s book Preference, Belief, and Similarity (2003).
3. **THE MODEL**

Consider a firm facing a production decision problem at the beginning of the period. It has to determine the level of input, $x$, which will produce an output $y = f_X(x) + e$ at the end of the period, where $e$ is a random variable with mean zero. The cost of production can be determined before the production is made, and expressed as $c(x)$, with $dc/dx = c'(x) > 0$. We make the assumption that the firm is in a competitive market and has no influence on the market price. At the end of the period, the firm can sell any amount of output it produces at a stochastic price $p$, where $E[p] = \mu$, $p = \mu + \sigma e_p$ and $e_p = 0$. The firm also has access to the forward market in which it can also choose to sell its output forward at a certain price $b$ at present. Let the amount sold in the forward market be $h$. If it turns out in the future that the hedged amount exceeds the real output, the firm can purchase the difference in the goods market at $p$ and sell in the forward market in order to complete the forward contract. For simplicity, we assume there is no time value of money involved in this problem. The firm is to maximize the expectation of the utility $U$, defined as a function of profit $\Pi$,

$$
\text{Max}_{x,h} \text{EU}(\Pi) = \int_{p} \int_{y} U[h(b - p) + px - c(x)] f_y(y \mid x) f_p(p) dy dp
$$

(1)

![Figure 1. Influence diagram for production and hedging decision: price and output uncertainty](image-url)
Under the assumption that $U(\Pi)$ is increasing, twice-differentiable and concave, that is, $U'(\Pi) > 0$ and $U''(\Pi) < 0$, the first-order to the maximization problem satisfies

$$E\{U'[p f'(x) - c'(x)]\} = 0 \quad (2)$$

$$E\{U'[b - p]\} = 0 \quad (3)$$

This result is simply achieved by differentiating $EU(\Pi)$ with respect to $x$ and $h$ respectively.

The problem rises when the utility function does not follow the ideal pattern. Non-linear programming methods may not be applicable since the concavity assumption is violated. Yet behavioral utilities have a lot of implications for real world organizational behaviors. The following section will demonstrate a numerical method to solve the general production decision problem with the help of decision analysis tools.

4. NUMERICAL EXAMPLES

4.1 Base Case Setup

The decision analysis formulation of the model is programmed in the Decision Programming Language (DPL) software. Solutions are obtained by choosing from different input and hedging levels to maximize expected utility. The numerical configuration of the example is presented below.

<table>
<thead>
<tr>
<th>Input Quantity</th>
<th>26</th>
<th>28</th>
<th>30</th>
<th>32</th>
<th>34</th>
<th>36</th>
<th>38</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of Output Quantity</td>
<td>2132</td>
<td>2296</td>
<td>2460</td>
<td>2624</td>
<td>2788</td>
<td>2952</td>
<td>3116</td>
</tr>
<tr>
<td>Stand Deviation of Output Quantity</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
</tbody>
</table>

Input cost rate = 166
Cash price at harvest ~ Lognormal (1.50588189, 0.028818849) (it is set such that Mean = 4.51)
Current Futures Price = 4.51 (unbiased)
Futures sold = 800, 1200, 1600, 2000, 2400, 2800, 3200, 3600, 4000
4.2 Incorporation of Behavioral Utility Functions

The above model is evaluated using three different kinds of utility functions.

4.2.1 Exponential Utility

The *exponential utility function* is the most basic and most commonly used utility function in uncertainty analysis. It possesses some desired features such as concavity and constant relative risk aversion (CARA).

In this example, it is defined as

\[ U_1(x) = 1 - e^{-x/\tau} \]  

(4)

where the risk tolerance \( \tau \) is set to be 6114.6.

![Figure 2. The exponential utility](image)

4.2.2 Piece-wise Linear Utility

The *piece-wise linear utility* is a utility with two linear components kinked at a reference point. The part below the point has a higher slope than that above it.

\[ U_2(x) = \begin{cases} 
(x - 6114.6)/SLOPE\_COEF + 6114.6, & x > 6114.6 \\
(x - 6114.6)\times SLOPE\_COEF + 6114.6, & x \leq 6114.6 
\end{cases} \]  

(5)

The slope coefficient is initially set to 2.

![Figure 3. The piece-wise linear utility](image)
4.2.3 Lose-Aversion Utility

The *loss-aversion utility* also has two parts separated by a reference point. The part above reference exhibits concavity while the part below is convex.

![Figure 4. The loss-aversion utility](image)

4.3 Results

The decision analysis model is run under three different utility functions. The optimal decisions are summarized in the table below:

<table>
<thead>
<tr>
<th>utility function</th>
<th>exponential</th>
<th>piece-wise</th>
<th>loss-aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal input decision</td>
<td>38</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>optimal futures bought</td>
<td>2800</td>
<td>4000</td>
<td>800</td>
</tr>
</tbody>
</table>

Figure 5 shows the risk profile associated with initial decisions in the base case (with exponential utility function). It shows that input decision of 38 is stochastic dominant over all the other alternatives:
In this example, the firm will choose to pursue maximum level of production regardless of its risk preference. However, it is important to note that the firm’s hedging decisions vary significantly across different utility functions.

5. DISCUSSION

In order to look into the firm’s optimal choices in various circumstances, some sensitivity analyses are performed.

5.1 Sensitivity to current futures price in the base case

The following graph is obtained by varying current futures price from 4.31 to 4.71 in steps of 0.02. Judging from the graph, the firm will choose the minimum level of future contracts (800 units) bought when current futures price is below 4.51 (the round dots), and the maximum hedging level (4000 units) when the futures price is above it (diamond dots). Only when the futures price is exactly 4.51 (the unbiased value) will the firm choose to purchase 2800 units of futures contract to partially hedge its output (the square dot).
5.2 **Two-way sensitivity in the piece-wise linear utility model**

Next, a two-way sensitivity analysis is performed by varying the slope coefficient and the input cost rate in the piece-wise linear utility model. Several discoveries are made. First, the firm will choose maximum input level at all times. Second, when the slope coefficient equals one, the utility function becomes linear. Risk preference does not play a role anymore and futures contract does not act as a useful hedging tool. The firm will become indifferent of any amount of hedging since it is risk neutral. Third, when the slope coefficient is above one, however, the firm’s optimal hedging decision solely depends on the input cost rate. It will choose to fully hedge when the cost per unit of input is below 132 or above 156, but will have to make various level of partial hedging decisions when the input cost rate is within the region. Finally, as long as the slope coefficient is above one, optimal production and hedging decisions will be independent of its value.
6. CONCLUSION AND FUTURE WORK

Production decision problems are very commonly in the modern manufacturing, agricultural and financial industries. This paper has demonstrated a generalized model and a decision analysis solution. It has taken into various aspects of the problem including price and output uncertainties, the use of forward contracts as hedging tools, and behavioral utility functions.

Hedging is a powerful tool to reduce the risk that the producer is facing. This paper discusses the firm’s optimal hedging decisions under different circumstances. According to the efficient-market hypothesis, the futures price is an unbiased estimate of the goods market cash price in the future. Therefore, the firm’s optimal hedging decision would be to partially hedge its output to maximize the expected utility. The model is capable of solving for the optimal decision given the sufficient market conditions.

In this study, organizational behavior in the form of behavior utilities is also incorporated together with the traditional exponential utility. All the three utilities functions are important, because empirical studies have already suggested that not all organizations are rational and behave in consistent with the risk-aversion assumption. This paper has similar findings compared to the traditional expected utility approach that the firm’s hedging decisions is strongly dependent on its risk preferences.

A limitation to the example is that the cost function is linear based on the competitive firm assumption. This leads to a smaller overall marginal cost than marginal revenue. This is the reason behind the firm’s decision to maximize input. A more exhaustive study will need to look into the economies and/or diseconomies of scale by using a different cost function. Nevertheless, the decision analysis model needs not to be changed.

One possible extension to the model is to put the production decision problem in a monopoly or oligopoly context. The influence of the firm’s size on market demand, together with the relationship between quantities and market price, need to be recognized. Under the new framework, the uncertainties in the market demand curve need be further specified to replace the competitive market assumption based on which the firm can sell any amount of output at a given price in order to make a more coherent representation of the market condition.

REFERENCES


Chavas Jean-Paul, and Holt M. T. (1990), Acreage Decisions Under Risk: The Case of Corn and


