UNCERTAINTY IN MEDICINE

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ABSTRACT

Uncertainty is inherent in medicine. Technological advance in clinical experiments changes the content of medical uncertainty and alter its contours, but uncertainty is never eliminated in this professional discipline. This is an introductory paper which explores the pertinent issue of uncertainty and medicine. This paper builds on the duality of Medicine not only as a Science, but also an Art. Emphasis is placed on understanding uncertainty from the position of physician and patient. The author also attempts to explore the uncertainty in medical decision-making with the use of Decision Trees, Bayes’s theorem and Epidemic Modeling with Reed-Frost Model & Kermack-McKendrick Model to provide the reader with a holistic approach to the interweaving subject of uncertainty and medicine.

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PROCEEDINGS AND FINDINGS

This paper has presented a holistic discussion of the uncertainty in the medical field. Beginning with a justification of the evidences of uncertainty inherent in medicine through studies on variations of medical practices within a geographical zone, the reader is then presented with a quantitative view of the cost to society as a result of medical uncertainty. Thereafter, the reader is demystified with regards to how decisions could be made in medicine using the analogy of decision tree models.

A solution to use posterior odds as a mean to avoid facing the confusion of $P(+) | D) = P(D | +)$ is also derived. Following which, an evaluation of socio-psychological studies done with patients on how their interaction with medical uncertainty is presented. A section on the difficulties of truth telling investigates the concerns for patients and physicians to share information which helps to diffuse uncertainty in diagnosis and treatment. After acquiring these knowledge, the reader is then presented with observations made from 2 similar studies conducted to find out the effect of uncertainty on physicians. Finally, modelling of disease is discussed to offer the reader a probabilistic approach to measure the uncertainty of the spread of

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an epidemic within a population. The reader might identify that locating and quarantine every single patient may not be entirely possible. Thus the uncertainty of the spread of an impending epidemic can now be demystified by acting with the result given by these probabilistic models.

SPECIAL FINDING FROM KM PROBABILISTIC MODEL

Kermack-McKendrick Model (KM) is the model which the author will be using to derive the population threshold beyond which an epidemic will propagate within the population. KM model also assume the relationship among the susceptibles, infectives and removals to be

\[
S_t + I_t + R_t = N. 
\]

KM model expects the number of susceptibles and infectives in the next interval to be

\[
\begin{align*}
S_{t+1} &= e^{a \log(1-p)I_t} S_t, \\
I_{t+1} &= (1 - e^{a \log(1-p)I_t}) S_t + bI_t,
\end{align*}
\]

\[
\therefore \text{we let } a = -\log(1-p), \text{we have}
\]

\[
\therefore b = \text{proportion of infectives who remain infectives at end of period}
\]

\[
\begin{align*}
S_{t+1} &= e^{-al} S_t, \\
I_{t+1} &= (1 - e^{-al}) S_t + bI_t, \\
R_{t+1} &= R_t + (1 - b)I_t,
\end{align*}
\]

Now we if \( I_0 = 1 \) person has the contagious disease in this population at time = 0.

\[
\begin{align*}
I_{t+1} &= (1 - e^{-al}) S_t + bI_t, \\
I_t &= (1 - e^{-al}) S_0 + bI_0, \\
\therefore \text{sub}\{I_0 = 1\} \\
I_t &= (1 - e^{-al}) S_0 + b(1) \\
I_t &= (1 - e^{-al}) S_0 + b \\
I_t &= (1 - e^{-al}) S_0 + b - 1 + 1 \\
I_t - 1 &= (1 - e^{-al}) S_0 + (b - 1) \\
\therefore \text{Equation is positive only if}
\end{align*}
\]

\[
S_0 > \frac{(1-b)}{1-e^{-a}} \quad \text{(Threshold } S^*)
\]

The above derivation shows that if the susceptible population size is larger than the threshold \( S^* \), then an epidemic will propagate, since the infective is able to gain new recruits. We can see
the numerator (1-b) as the probability of an infective not surviving to the next sampling period. Improved heath services and treatment of the population by public health authority can increase this number! The denominator (1 – e^{-r}) is the probability of being infected. This number can be reduced by reducing contact between individuals, an example will be quarantine exercise during the SARs epidemic. This derivation using the defining equation of KM model provide the threshold population value as a useful criteri on for controlling a contagious disease. The findings from KM model complement the findings provided by the random structure inherent in the Reed Frost Model, which is also investigated in detail in this paper. In addition, stochastic models explored at the conclusion of this paper provide another framework for analysing the uncertainty in medical epidemics.

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